|  | Ques | Answer | Marks | Guidance |
| :---: | :---: | :---: | :---: | :---: |
| 1 | (i) | $\begin{aligned} & \frac{1}{\sqrt[3]{1-2 x}}=(1-2 x)^{-1 / 3} \\ & =1+\left(-\frac{1}{3}\right)(-2 x)+\frac{\left(-\frac{1}{3}\right)\left(-\frac{4}{3}\right)}{2!}(-2 x)^{2}+\ldots \\ & =1+\frac{2}{3} x+\frac{8}{9} x^{2}+\ldots \end{aligned}$ <br> Valid for $-\frac{1}{2}<x<\frac{1}{2}$ or $\|x\|<\frac{1}{2}$ | B1 <br> M1 <br> B1 <br> B1 <br> B1 <br> [5] | $n=-1 / 3$. See below $\mathbf{S C}$ for those with $n=1 / 3$ <br> All three correct unsimplified binomial coefficients (not nCr ) soi condone absence of brackets only if it is clear from subsequent work that they were assumed $1+(2 / 3) x+\ldots$ www <br> $(8 / 9) x^{2}$ www in this term <br> If there is an error, in say, the third coefficient of the expansion then M0B1B0 is possible <br> SC For $n=1 / 3$ award B1 for $1-(2 / 3) x$ and B1 for $-(4 / 9) x^{2}$ (so $\max 2$ out of the first 4 marks) <br> Independent of expansion. Accept, say, $-1 / 2<\|x\|<1 / 2$ or $-1 / 2 \leq x<1 / 2$ (must be strict inequality for $+1 / 2$ ) |
| 1 | (ii) | $\begin{aligned} & \begin{aligned} \frac{1-3 x}{\sqrt[3]{1-2 x}} & =(1-3 x)\left(1+\frac{2}{3} x+\frac{8}{9} x^{2}+\ldots\right) \\ & =1+\frac{2}{3} x+\frac{8}{9} x^{2}-3 x-2 x^{2}+\ldots \end{aligned} \\ & =1-\frac{7}{3} x-\frac{10}{9} x^{2}+\ldots \end{aligned}$ | M1 <br> A1ft <br> A1 <br> [3] | Use of $(1-3 x) \times$ their $\left(1+(2 / 3) x+(8 / 9) x^{2}+\cdots\right)$ and attempt at removal of brackets (condone absence of brackets but must have two terms in $x$ and two terms in $x^{2}$ ) <br> Correct simplified expansion following their expansion in (i). This mark is dependent on scoring both M marks in (i) and (ii) <br> cao or B3 www in either part <br> SC following either M0 or M1, B1 for either a or b correct |


| Question |  | Answe | Marks | Guidance |
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| 2 |  | $(4+x)^{\frac{3}{3}}=4^{\frac{3}{2}}\left(1+\frac{1}{4} x\right)^{\frac{3}{2}}$ | M1 | dealing with the ' 4 'to obtain $4^{3 / 2}\left(1+{ }_{4}^{x}\right)^{3 / 2}$ <br> (or expanding as $4^{3 / 2}+\frac{3}{2} 4^{1 / 2} x+\binom{3}{2}\binom{1}{2} 4^{-1 / 2} x^{2}+\ldots$ and having all the powers of 4 correct) |
|  |  | $=8\left(1+\frac{3}{2}\left(1 x_{4}^{1} x\right)+{ }_{2}^{3} \cdot \frac{1}{2} \cdot \frac{1}{2!}\left(1_{4}^{1} x\right)^{2}+\ldots\right.$ | M1 | correct binomial coeffs for $n=3 / 2$ ie 1, 3/2,3/2.1/2.1/2! Not nCr form Indep of coeff of $x$ <br> Indep of first M1 |
|  |  | $=8+3 x$ | A1 | $8+3 x \quad$ www |
|  |  | $+3 / 16 x^{2}$ | A1 | $\ldots+3 / 16 x^{2} \quad \text { www }$ <br> Ignore subsequent terms |
|  |  | Valid for $-4<x<4$ or $\|x\|<4$ | B1 | accept $\leq \mathrm{s}$ or a combination of $<$ and $\leq$, but not $-4>x>4,\|x\|>4$, or say $-4<x$ <br> condone $-4<\|x\|<4$ <br> Indep of all other marks |
|  |  |  | [5] | Allow MR throughout this question for $n=m / 2$ where $m \in \mathrm{~N}$, and m odd and then -1 MR provided it is at least as difficult as the original. |


| Question |  | Answer | Marks | Guidance |
| :---: | :---: | :---: | :---: | :--- |
| 3 | (i) | $\begin{array}{ll}\frac{x}{(1+x)(1-2 x)}=\frac{A}{1+x}+\frac{B}{1-2 x} \\ \Rightarrow \quad x=A(1-2 x)+B(1+x) \\ x=1 / 2 \Rightarrow 1 / 2=B(1+1 / 2) \Rightarrow B=1 / 3 \\ x=-1 \Rightarrow-1=3 A \Rightarrow A=-1 / 3\end{array}$ | M1 | $\begin{array}{l}\text { A1 } \\ \text { expressing in partial fractions of correct form (at any stage) and } \\ \text { attempting to use cover up, substitution or equating coefficients } \\ \text { Condone a single sign error for M1 only. } \\ \text { www cao } \\ \text { A1 }\end{array}$ |
| www cao |  |  |  |  |
| (accept $\mathrm{A} /(1+\mathrm{x})+\mathrm{B} /(1-2 \mathrm{x}), \mathrm{A}=-1 / 3, \mathrm{~B}=1 / 3$ as sufficient for |  |  |  |  |
| full marks without needing to reassemble fractions with numerical |  |  |  |  |
| numerators) |  |  |  |  |$]$


| Question |  | Answer | Marks | Guidance |
| :---: | :---: | :---: | :---: | :---: |
| 3 | (ii) | $\begin{aligned} & \frac{x}{(1+x)(1-2 x)}=\frac{-1 / 3}{1+x}+\frac{1 / 3}{1-2 x} \\ & =\frac{1}{3}\left[(1-2 x)^{-1}-(1+x)^{-1}\right] \\ & =\frac{1}{3}\left[1+(-1)(-2 x)+\frac{(-1)(-2)}{2}(-2 x)^{2}+\ldots-\left(1+(-1) x+\frac{(-1)(-2)}{2} x^{2}+\ldots\right)\right] \\ & =\frac{1}{3}\left[1+2 x+4 x^{2}+\ldots-\left(1-x+x^{2}+\ldots\right)\right] \end{aligned}$ $=\frac{1}{3}\left(3 x+3 x^{2}+\ldots\right)=x+x^{2}+\ldots \text { so } a=1 \text { and } b=1$ | M1 <br> A1 <br> A1 <br> A1 | correct binomial coefficients throughout for first three terms of either $(1-2 x)^{-1}$ or $(1+x)^{-1}$ oe ie $1,(-1),(-1)(-2) / 2$, not nCr form. Or correct simplified coefficients seen. $1+2 x+4 x^{2}$ <br> $1-x+x^{2} \quad$ (or $1 / 3 /-1 / 3$ of each expression, ft their $A / B$ ) <br> If $k\left(1-x+x^{2}\right)$ (A1) not clearly stated separately, condone absence of inner brackets (ie $1+2 x+4 x^{2}-1-x+x^{2}$ ) only if subsequently it is clear that brackets were assumed, otherwise A1A0. <br> [ie $-1-x+x^{2}$ is A0 unless it is followed by the correct answer] Ignore any subsequent incorrect terms <br> or from expansion of $x(1-2 x)^{-1}(1+x)^{-1}$ <br> www cao |
|  |  | OR $\begin{aligned} & x(1-x-2\left.x^{2}\right) \\ &=x\left(1-\left(x+2 x^{2}\right)\right) \\ &=x\left(1+x+2 x^{2}+(-1)(-2)\left(x+2 x^{2}\right)^{2} / 2+\ldots \ldots \ldots\right) \\ &=x\left(1+x+2 x^{2}+x^{2} \ldots \ldots \ldots\right) \\ &=x+x^{2} \ldots . . \text { so } a=1 \text { and } b=1 \end{aligned}$ | M1 <br> A2 <br> A1 | correct binomial coefficients throughout for (1-( $\mathrm{x}+2 \mathrm{x}^{2}$ )) oe (ie $1,-1$ ), at least as far as necessary terms ( $1+\mathrm{x}$ ) (NB third term of expansion unnecessary and can be ignored) <br> $x(1+x)$ www <br> ww cao |
|  |  | Valid for $-1 / 2<x<1 / 2$ or $\|x\|<1 / 2$ | B1 [5] | independent of expansion. Must combine as one overall range. condone $\leq \mathrm{s}$ (although incorrect) or a combination. Condone also, say $-1 / 2<\|x\|<1 / 2$ but not $x<1 / 2$ or $-1<2 x<1$ or $-1 / 2>x>1 / 2$ |


$5 \begin{aligned} \quad(1+3 x)^{\frac{1}{3}} & =1+\frac{1}{3}(3 x)+\frac{\frac{1}{3} \cdot\left(-\frac{2}{3}\right)}{2!}(3 x)^{2}+\ldots \\ & =1+x-x^{2}+\ldots\end{aligned}$

Valid for $-1 \leq 3 x \leq 1$
$-1 / 3 \leq x \leq 1 / 3$

## M1 <br> correct binomial coefficients

A1
A1
$1+x \ldots$
$\ldots-x^{2}$

M1 $\quad$ or $|3 x| \leq 1 \quad$ oe
A1 or $|x| \leq 1 / 3$
or $|x| \leq 1 / 3$
(correct final answer scores M1A1)
ie $1,1 / 3,(1 / 3)(-2 / 3) / 2$ not $n C r$ form simplified www in this part
simplified www in this part, ignore subsequent terms using $(3 x)^{2}$ as $3 x^{2}$ can score M1B1B0 condone omission of brackets if $3 x^{2}$ is used as $9 x^{2}$ do not allow MR for power 3 or $-1 / 3$ or similar
condone inequality signs throughout or say < at one end and $\leq$ at the other
condone $-1 / 3 \leq|x| \leq 1 / 3, \quad x \leq 1 / 3$ is M0A0 the last two marks are not dependent on the first three

$$
\begin{aligned}
& \text { 6(i) } \frac{3+2 x^{2}}{+(1 x)(1-4) 1}=\frac{A}{+x}+\frac{B}{+(k) \quad 4^{2} 4}{ }^{2}-\frac{C}{x} \\
& \Rightarrow 3+2 x^{2}=A(1+x)(1-4 x)+B(1-4 x)+C(1+x)^{2} \\
& x=-1 \Rightarrow 5=5 B \Rightarrow B=1 x=1 / 4 \Rightarrow{ }_{3}{ }^{1}= \\
& { }^{25}{ }_{C} \Rightarrow C=2 \quad \overline{8} \quad \overline{16} \\
& \text { coeff }^{\mathrm{t}} \text { of } x^{2}: 2=-4 A+C \Rightarrow A=0 .
\end{aligned}
$$

$$
\text { (ii) }(1+x)^{-2}=1+(-2) x+(-2)(-3) x^{2} / 2!+\ldots
$$

$$
=1-2 \quad x+3 x^{2}+\ldots
$$

$$
(1-4 x)^{-1}=1+(-1)(-4 x)+(-1)(-2)(-4 x)^{2} / 2!+\ldots
$$

$$
=1+4 x+16 x^{2}+\ldots
$$

$$
\frac{3+2 x^{2}}{+(\mathrm{k})^{2}(1-4 x)}
$$

$$
\approx 1-2 x+3 x^{2}+2\left(1+4 x+16 x^{2}\right)=3+6 x+35 x^{2}
$$M1

M1 Clearing fractions (or any 2 correct equations)
$B \quad=\quad 1 \quad$ www
$C=2 \mathrm{www}$
$A=0$ needs justification

1

## 1

Binomial series (coefficients unsimplified - for either
or $\left(3+2 x^{2}\right)(1+x)^{-2}(1-4 x)^{-1}$ expanded

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| $\left.\left.\left.\begin{array}{rl} 7 & \sqrt{4+2 x}=2\left(1+\frac{1}{2} x\right)^{\frac{1}{2}} \\ & =2\left\{1+\frac{1}{2} \cdot\left(\frac{1}{2} x\right)+\frac{\stackrel{1}{2} \cdot\left(-\frac{1}{2}\right)}{2!}\left(\frac{1}{2} x^{2}+\frac{{ }^{2}}{2} \cdot\left(-\frac{1}{2}\right) \cdot\left(-\frac{3}{2}\right)\right.\right. \\ 3! \\ 2 \\ 2 \\ 2 \end{array}\right)^{3}+\ldots\right\}\right)$ | M1 <br> M1 <br> A2,1,0 | Taking out 4 oe correct binomial coefficients $\frac{1}{4} x,-\frac{1}{32} x^{2},+\frac{1}{128} x^{3}$ |
| :---: | :---: | :---: |
| $=\left(2+\frac{1}{2} x-\frac{1}{16} x^{2}+\frac{1}{64} x^{3}+\ldots\right)$ | A1cao |  |
| Valid for $-2<x<2$. | B1cao [6] |  |

